## Lionfish Matrix Population Model

A matrix population model for lionfish was developed by Morris et al. (2011) to investigate potential approaches for controlling the invasive species. Lionfish start reproducing in one year. Once they mature, they are very fecund, releasing a large number of eggs every three days according to the study. These life history strategies make them very successful invasive species. Morris. et al. (2011) took parameters from other studies (Table 1) and produced a three-stage (larvae, juvenile, and adult) population matrix (with one-month time step):

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We refers this model as model 1 (M1).

Table 1. Estimated demographic parameters and calculated matrix population model elements. Modified from Tables 1 & 2 in Morris et al. (2011).

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| Estimated demographic parameters | | |
| Parameters | **Value** | **Units** |
| Larval mortality (*ML*) | 0.350 | days-1 |
| Adult mortality (*MA*) | 0.052 | months-1 |
| Juvenile mortality (*MJ*) | 0.165 | months-1 |
| Proportion female (*ρ*) | 46% |  |
| Larval duration (*DL*) | 30 | days |
| Egg mortality (*ME*) | 0.310 | days-1 |
| Fecundity (*f*) | 194,577 | Eggs months-1 female-1 |
| Egg duration (*DE*) | 3 | days |
| Calculated matrix model elements | | |
| Element | **Value** | **Mathematical Notation** |
| *PA* | 0.949 |  |
| *PJ* | 0.777 |  |
| *GL* | 0.00003 |  |
| *GJ* | 0.071 |  |
| *RA* | 35315 |  |

We found two major problems in the construction of the matrix elements. First, the model is a post-breeding census so that the survival of adults have to be incorporated into the fertility term *RA*. Because the fertility term include 3-day survival of eggs, the fertility rate should be multiplied by 27-day survival of adults assuming one month consists of 30 days. Second, lionfish start reproducing in one year (i.e. 12 months). If so, they should spend 10 months on average in the juvenile stage rather than 12 months because individuals spend one month in larval stage, and another month is included in the fertility term. Therefore, the coefficients in *PJ* and *GJ* should be 9/10 and 1/10, respectively. These corrections were incorporated into model 2 (M2) and model 3 (M3), respectively. We are aware that the fertility rate calculation still has a problem. They release eggs every three days. However, incorporating three-day birth intervals is difficult because it also affects the duration in the larval stage. Once they hatch, they are in a larval stage for 30 days, but the ones that were born early would have spent almost 30 days before they appear in the larval stage. For now, we assume they reproduce every 30 days.

Another way of calculating *PJ* and *GJ* were also incorporated into model 4 (M4) in addition to the two corrections incorporated into M2 and M3.





where  (stage specific survival rate). This assumes a fixed duration in the juvenile stage.

The above calculation is only accurate when λ is close to 1. When it is far from 1, *PJ* and *GJ* calculations need to be modified (M5):



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In this method, an arbitrarily λ was set to 1, initially. Then, the population matrix was obtained, and λ was calculated. Then, the population matrix was modified using the calculated λ, and the new λ was calculated again. This process was repeated until λ converges to some value.

Finally, the Leslie matrix was constructed based on the parameters in the table (model 6, M6). This model consisted of 12 age classes so that first offspring are observed at the end of 12th month after birth.

Using the six models, asymptotic population growth rate λ, stable stage distribution, reproductive value, and sensitivity of the population growth rate to stage specific survival rate and fecundity (*f*) were calculated. Reproductive values for all models were scaled so that the reproductive value of first stage is 1. For calculating the stable stage distribution, reproductive value, and sensitivity, under M6 for the juvenile stage, corresponding values for age-classes 2 to 11 were summed. For calculating the sensitivity of lambda under M5, numerical differentiation was used.



Figure Asymptotic population growth rates of M1-M6.



Figure . Stable stage distribution of Juvenile stage and adult stage for the six models.



Figure . Reproductive value of juveniles and adults under the six models.



Figure . Sensitivity of asymptotic population growth rate to survival and fertility rates.



Figure Elasticity of lambda to survival and reproduction